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As a generalization of the problem of S. P. Timoshenko (Ref. 1) of the buckling of an infinitely long cylindrical shell subject to torsion, let us consider the conditions for stability of that shell under the combined action of torsion and internal pressure. The solution will be obtained in closed form.

Making use of the equilibrium equations found in Ref. 1 and using the notation of the author of that book for the displacements u, v, and w, we get the following equations:

$$
\begin{aligned}
&a\frac{\partial^2 u}{\partial x^2} + \frac{a(1+\mu)}{2}\frac{\partial^2 v}{\partial x\,\partial\theta} - \mu a\frac{\partial w}{\partial x} - a\varphi_1\left(\frac{\partial^2 v}{\partial x\,\partial\theta} - \frac{\partial w}{\partial x}\right) \\
&\qquad + \frac{(1-\mu)}{2}\frac{\partial^2 u}{\partial\theta^2} + a\varphi\left(\frac{\partial^2 u}{\partial x\,\partial\theta} - a\frac{\partial^2 v}{\partial x^2}\right) = 0 \\[2ex]
&a\frac{(1+\mu)}{2}\frac{\partial^2 u}{\partial x\,\partial\theta} + \frac{a^2(1-\mu)}{2}\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial\theta^2} - \frac{\partial w}{\partial\theta} + \alpha\left[\frac{\partial^2 v}{\partial\theta^2} + \frac{\partial^3 w}{\partial\theta^3}\right. \\
&\qquad \left. + a^2\frac{\partial^3 w}{\partial x^2\,\partial\theta} + a^2(1-\mu)\frac{\partial^2 v}{\partial x^2}\right] + a^2\varphi_2\frac{\partial^2 v}{\partial x^2} + 2a\varphi\left(\frac{\partial^2 v}{\partial x\,\partial\theta} - \frac{\partial w}{\partial x}\right) = 0 \qquad (1) \\[2ex]
&\mu a\frac{\partial u}{\partial x} + \frac{\partial v}{\partial\theta} - w - \alpha\left[\frac{\partial^3 v}{\partial\theta^3} + (2-\mu)a^2\frac{\partial^3 v}{\partial x^2\,\partial\theta} + a^4\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial\theta^4}\right. \\
&\qquad \left. + 2a^2\frac{\partial^4 w}{\partial x^2\,\partial\theta^2}\right] + \varphi_1\left(w + \frac{\partial^2 w}{\partial\theta^2}\right) + a^2\varphi_2\frac{\partial^2 w}{\partial x^2} + 2a\varphi\left(\frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x\,\partial\theta}\right) = 0
\end{aligned}
$$

where:

$$\varphi = \frac{\tau}{E}(1-\mu^2),\ \varphi_2 = \frac{N_x}{Eh}(1-\mu^2),\ \varphi_1 = \frac{qa}{Eh}(1-\mu^2),\ \alpha = \frac{1}{12}\left(\frac{h}{a}\right)^2$$

For infinitely long cylindrical shells, system (1) has the solution:

$$u = A\cos\left(\frac{\lambda x}{a} - n\theta\right),\quad v = B\cos\left(\frac{\lambda x}{a} - n\theta\right),\ w = C\sin\left(\frac{\lambda x}{a} - n\theta\right) \tag{2}$$

Substitute these expressions for displacements in (1) to get:

$$A\left[\lambda n\varphi - \lambda^2 - \frac{(1-\mu)}{2}n^2\right] + B\left[\frac{(1+\mu)}{2}\lambda n + \lambda^2\varphi - \lambda n\varphi_1\right] + C\left[\lambda\varphi_1 - \lambda\mu\right] = 0$$

$$A\frac{(1+\mu)}{2}\lambda n + B\left[2\lambda n\varphi - \lambda^2\varphi_2 - n^2(1+\alpha) - \frac{1-\mu}{2}\lambda^2(1+2\alpha)\right] + C\left[n + \alpha n^3 + \alpha\lambda^2 n - 2\lambda\varphi\right] = 0 \tag{3}$$

$$A\mu\lambda + B\left[2\lambda\varphi - n - \alpha n^3 - (2-\mu)\lambda^2 n\alpha\right] + C\left[1 + \alpha(\lambda^2+n^2)^2 - 2\lambda n\varphi + \varphi_1(n^2-1) + \lambda^2\varphi_2\right] = 0$$

Equate the determinant of the system of linear equations (3) to zero and neglect higher order terms containing α^2, φ^2, φ_1^2, $\alpha\varphi$, etc. to obtain:

$$
\begin{aligned}
&2\varphi(\lambda n^5 - \lambda n^3 + 2\lambda^3 n^3 - \lambda^3 n + \lambda^5 n) \\
&-\varphi_1\left[\lambda^4 n^2 + 2\lambda^2 n^4 - 2\lambda^2 n^2 - \lambda^4(1-\mu) + n^6 - n^4\right] \\
&-\varphi_2\left[\lambda^2 n^2 + 2\lambda^4 n^2 + \lambda^2 n^4 + 2\lambda^4(1+\mu) + \lambda^6\right] \\
&-\left\{\lambda^4(1-\mu^2) + \alpha\left[2\lambda^4(1-\mu^2) + (\lambda^2 - n^2)^4 + (3+\mu)\lambda^2 n^2\right.\right. \\
&\left.\left.-(2+\mu)(3-\mu)\lambda^4 n^2 - (7+\mu)\lambda^2 n^4 - 2n^6 + n^4\right]\right\} = 0
\end{aligned}
\tag{4}
$$

In the particular case considered by S. P. Timoshenko, where $\varphi_1 = \varphi_2 = 0$, equation (4) simplifies considerably and, by assuming that λ is a small parameter and letting n = 2, one easily determines the minimum value of the critical shear stress. However, with combined torsion and internal pressure the minimum values of the function φ occur with significantly greater values of λ and n and only a limited amount of simplification is possible in equation (4). In this case we will investigate equation (4) numerically.

Consider an infinitely long closed tube, let $\varphi_2 = \frac{1}{2}\varphi_1$, and neglect the higher order terms containing $\alpha\lambda^4$, αn^4, $\alpha\lambda^2 n^2$, etc. Equation (4) may now be written in the trinomial form:

4

$$\varphi = L(\lambda, n)\varphi_1 + M(\lambda, n)\alpha + N(\lambda, n) \qquad (5)$$

where:

$$L(\lambda, n) = \frac{n^6 + n^4(\frac{5}{2}\lambda^2 - 1) + n^2(2\lambda^4 - \frac{3}{2}\lambda^2) + \frac{1}{2}\lambda^6 + 2\mu\lambda^4}{2\lambda n\left[n^4 + n^2(2\lambda^2 - 1) + \lambda^4 - \lambda^2\right]}$$

$$M(\lambda, n) = \frac{(\lambda^2 + n^2)^2}{2\lambda n\left[n^4 + n^2(2\lambda^2 - 1) + \lambda^4 - \lambda^2\right]}$$

$$N(\lambda, n) = \frac{\lambda^4(1 - \mu^2)}{2\lambda n\left[n^4 + n^2(2\lambda^2 - 1) + \lambda^4 - \lambda^2\right]}$$

Fix the magnitudes of the parameters α and φ_1. By successively assigning values to n as λ takes on the values $\lambda_1, \lambda_2, \ldots$, we may find relative minimum values of the function φ corresponding to $n = n_1, n_2, \ldots$ and may finally determine the specific minimum value of the critical shear stress.

The results of these calculations of the minimum values of the parameter φ as a function of φ_1 from formula (5) are, for $\mu = 0.3$:

$\varphi_1 \cdot 10^3$	$\varphi \cdot 10^3$				
	$\frac{a}{h} = \infty$	$\frac{a}{h} = 1200$	$\frac{a}{h} = 600$	$\frac{a}{h} = 300$	$\frac{a}{h} = 150$
0	0	0.0055	0.0155	0.0437	0.1070
0.05	0.035	0.148	0.224	0.330	0.479
0.10	0.071	0.222	0.316	0.465	0.672
0.20	0.141	0.335	0.453	0.645	0.936
0.40	0.283	0.511	0.669	0.906	1.305
0.60	0.424	0.658	0.852	1.125	1.624
0.80	0.566	0.803	1.027	1.332	1.910
1.00	0.707	0.950	1.189	1.531	2.156

Analysis of these results shows that the critical external load increases substantially in the presence of internal pressure.

In conclusion, note that for infinitely thin shells under the action of internal pressure the relation $\varphi = f(\varphi_1)$ is linear as is evident from the table. It is possible to obtain the same result, namely $\varphi = (1/\sqrt{2})\,\varphi_1$, from the condition that buckling under shear is possible when the smaller of the principal stresses vanishes.

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Reference

1. Timoshenko, S. P., Stability of Elastic Systems, GITTL, Moscow, 1955.

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